# Indian Statistical Institute, Bangalore Centre. End-Semester Exam : Discrete Mathematics I - B2 

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Max. points : 10.
Time Limit : 1.5 hours.
Give complete proofs. Please cite/quote appropriate results from class. You are also allowed to use results from other problems in the question paper.All questions carry equal points.

## Attempt any two questions only. If more than two questions are attempted, only the first two will be corrected.

1. Let $G$ be an $n$-vertex graph. If every vertex has degree at least $(n-1) / 2$ then $G$ is connected.
2. Let $0<a_{1}<\ldots<a_{s r+1}$ be $s r+1$ integers. Prove that we can select either $s+1$ of them, no one of which divides any other, or $r+1$ of them with each dividing the following one.
3. Let $G$ be a bi-partite graph with partition $V=A \sqcup B$. Let $\max _{v \in B} d_{v} \leq$ $\min _{u \in A} d_{u}$ i.e., the minimum degree of vertices in $A$ is at least that of the maximum degree of vertices in $B$. Show that there exists a complete matching on $A$.
